Topics in Macroeconomics

Mock Class Exam 1: Solutions

Question 1(a): Long-Run Wages & Prices in Augmented Solow

Instructions: Answer each short question based on the concepts covered in the course. The arguments should be primarily **verbal** (though you can use formulas or graphs if you wish). Be concise, write **no more than about half a page** per question.

Question:

In a standard decentralised and augmented **Solow model** (with population and technology growth), how do the **wage** and the **rental rate of capital** move in the **long run**?

Provide an *intuition* for these results.

Q1(a): Long-Run Wages & Prices in Augmented Solow

Recall: Factor prices in decentralized Solow model (Tutorial 2 Q1). From firms' profit maximization:

$$R_t = \underbrace{\alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha}}_{= \text{MPK}_t}, \qquad W_t = \underbrace{(1-\alpha) K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha}}_{= \text{MPL}_t}. \qquad \textit{Hint}\text{: Write in terms of } \tilde{k}_t = \frac{K_t}{A_t L_t}!$$

Result: Wage

On the balanced growth path, the **wage grows at the rate of technology**, g:

$$W_t \to W_t^* = A_t(\tilde{k}^*)^{\alpha} \propto A_t \quad \Longrightarrow \quad \frac{W_{t+1}}{W_t} = 1 + g.$$

Result: Rental Rate

The **rental rate of capital is constant** in the long run:

$$R_t \to R^* = \alpha\left(\tilde{k}^*\right)^{\alpha-1} = \text{const.}$$

where $\tilde{k}^* = K/(AL)$ is the steady-state effective capital-labour ratio.

Intuition

Wage ↑ at rate g: Capital K_t grows faster (rate $g + n + gn$) than labour L_t (rate n), so $\text{MPL}_t = W_t^*$ is growing. Labour supply is not endogenous, so wages are pushed up in line with A_t.

Rental rate constant: The supply of capital is *endogenous* — it rises with output (investment is a constant fraction of output). Demand and supply of capital grow at the same rate, so R_t stays constant.

Clear example: (constant L, growing A_t):
- fixed labour supply ⇒ rising labour demand ⇒ W_t ↑;
- capital supply rises with output ⇒ R_t unchanged.

Question 1(b): Capital transition in the Ramsey Model

Question:

Consider the standard **Ramsey model** developed in the course. Suppose that in the initial period, the capital stock is above its steady state value, $K_0 > K^*$.

Provide an intuition for why the economy will not reach its steady state within a period, meaning that $K_1 \neq K^*$.

Q1(b): Capital transition in the Ramsey Model

Setting

- Capital starts **above** steady state: $K_0 > K^*$
- Capital converges to K^*, so it steadily **decreases** over time
- But it does **not** jump to K^* in one period

Hint: Explain basic logic about transitional dynamics in Solow/Ramsey at the start of your answer! That is, model always converges to steady state or BGP.

1. Counterfactual: What if $K_1 = K^*$?

If $K_1 = K^*$, consumption would be **constant** at C^* from period 1 onward — but **preceded by a large drop** from period 0 (when income is high).

2. Why the Household Dislikes This

Household has a **concave** period utility function:

$$u(C_t) = \frac{C_t^{1-\theta} - 1}{1 - \theta}$$

Features *diminishing marginal returns to consumption*:

$$u''(C) < 0.$$

This means the household **prefers smooth consumption** over time.

- Strength of preference is measured by θ.

A large bump in C_0 followed by lower but constant consumption is suboptimal — household does **better** by letting C fall gradually $\rightarrow$ K also falls gradually:

$$K_0 > K_1 > K_2 > \cdots \rightarrow K^*.$$

Question 1(c): Transitional dynamics in augmented Solow

Question:

Consider a standard augmented **Solow model** with positive long-run technological growth, $g > 0$, and a constant population.

Is it possible for such a model economy to exhibit the following **pattern in output**, Y_t, over time t:

$$Y_0 > Y_1 > Y_2 \quad \text{and then} \quad Y_2 < Y_3 < \cdots < Y_\infty \text{ ?}$$

Explain your reasoning.

Q1(c): Transitional dynamics in augmented Solow

Long-Run Behaviour

Step 1: Augmented Solow → convergence to a **BGP**.
Step 2: On the BGP, output grows at rate $g > 0$, so

$$Y_2 < Y_3 < \cdots < Y_\infty$$

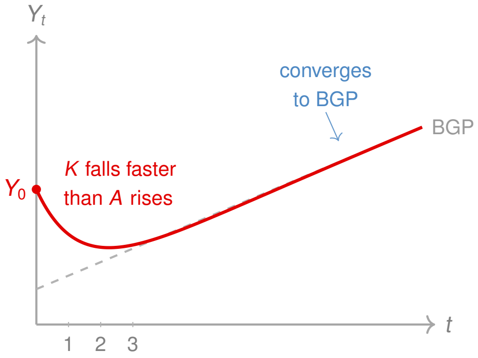
is **always** the case in the long run. ✓

Initial Fall: When Is It Possible?

Output can **initially fall** if $\tilde{k}_0 = K_0/(A_0 L_0)$ is **sufficiently above** its BGP level.

- Investment $I = sY$ initially **not enough to offset depreciation** $\Rightarrow K$ falls
- If K falls enough to *outweigh* rise in A_t, then Y_t falls

Once $K/AL \to \tilde{k}^*$, technology growth dominates and Y_t resumes growing.



Question 1(d): Misallocation and TFP across countries

Question:

Briefly describe in words how **misallocation** may be responsible for the cross-country variation in total factor productivity.

Q1(d): Misallocation and TFP across countries

Step 1: Defining misallocation and its link to TFP

A given stock of **aggregate factors** generates more or less output depending on how physical and human capital are **assigned at the micro level**. Through the lens of the aggregate production function, misallocated economies show up as having **lower TFP**.

Step 2: Evidence and Examples

There is substantial evidence that developing economies could raise productivity by reassigning factors. **Two leading examples**:

- **Across firms:** Reallocation of capital and labour from less productive to **more productive** firms. E.g. *small* to *large*, or from *financially unconstrained* to *financially constrained* firms — i.e. firms that cannot raise sufficient capital to fund lucrative projects. (see Hsieh & Klenow, 2009)
- **Across sectors:** Many developing countries would raise GDP by reallocating labour *away* from agricultural toward non-agricultural activities.

  Reason: Agriculture is much less productive (relative to other sectors) than in advanced economies.

Question 1(e): UK Nominal GDP Growth

Question:

The **UK nominal GDP** increased from **GBP 1,549 billion** in 2009 to **GBP 2,218 billion** in 2019.

Use the formula $x_t = x_0(1+g)^t$ to compute the **average annual growth rate**.

To what extent does that number reflect the average growth rate in **real GDP per worker** over that period? Explain.

Q1(e): UK Nominal GDP Growth

Step 1: Compute the growth rate

Apply the formula with $x_0 = 1549$, $x_{10} = 2218$, $t = 10$:

$$2218 = 1549\,(1+g)^{10}$$

$$(1+g)^{10} = \frac{2218}{1549} = 1.432$$

Take logs:

$$10\log(1+g) = \log(1.432)$$

$$1 + g = \exp\left\{\tfrac{1}{10}\log(1.432)\right\}$$

$$\implies \quad g \approx 0.037 \quad (3.7\%\text{ per year})$$

Note: You would probably have liked to have a calculator!

Step 2: Is this real GDP per worker?

No. To get real GDP per worker we would need to subtract two further components:

- **Price growth:** Nominal GDP rises with the price level. We need the GDP deflator to convert to real terms.
- **Workforce growth:** Real GDP per worker requires dividing by the number of workers.

Both the price level and the number of workers **grew** over 2009–2019, each "biasing" nominal GDP growth upward relative the growth rate of real GDP per worker.