$$Y_t = K_t^{\alpha}(AL_t)^{1-\alpha}$$

(a) $R_t = MPK$ $\qquad W_t = MPL$.

$$\boxed{R_t = MPK} = \frac{\partial Y_t}{\partial K_t} = \alpha K_t^{\alpha-1}(AL_t)^{1-\alpha} \parallel \quad \alpha A^{1-\alpha}\left(\frac{K_t}{L_t}\right)^{\alpha-1}$$

k_t

$$\boxed{W_t = MPL} = \frac{\partial Y_t}{\partial L_t} = (1-\alpha) K_t^{\alpha} A^{1-\alpha} L_t^{-\alpha} \parallel$$

Assumption 1

(b) $\Pi_t = Y_t - C = Y_t - R_tK_t - W_tL_t$

$$= K_t^{\alpha}(AL_t)^{1-\alpha} - \underbrace{\alpha K_t^{\alpha-1}(AL_t)^{1-\alpha}}_{mpk,\ R_t} \cdot K_t - \underbrace{(1-\alpha)K_t^{\alpha}A^{1-\alpha}L_t^{-\alpha}}_{mpL,\ W_t} \cdot L_t$$

$$= K_t^{\alpha}(AL_t)^{1-\alpha} - \alpha \underbrace{K_t^{\alpha}(AL_t)^{1-\alpha}}_{Y_t} - (1-\alpha)\underbrace{K_t^{\alpha}(AL_t)^{1-\alpha}}_{Y_t}$$

Assumption 2

$$= \underbrace{K_t^{\alpha}(AL_t)^{1-\alpha}}_{Y_t} - 1 \cdot \underbrace{K_t^{\alpha}(AL_t)^{1-\alpha}}_{Y_t}$$

$= 0$.

Assumptions : ①. perfectly competitive market.

② Constant return to scale.

In equilibrium. L_t .... demand. $\quad L_t = L$

L .... supply.

exogeneous.

HH's total income: $R_tK_t + W_tL = Y_t$

$= R_tK_t + W_tL_t$

(c) $W_t = (1-\alpha) K_t^{\alpha} A^{1-\alpha} L^{-\alpha}$

① $K \uparrow$, $W_t \uparrow$.

K, L. complementary in production.

$K\uparrow$, L productive, $MPL \uparrow$, $W_t \uparrow$

② $A\uparrow$, $W_t \uparrow$

A ... TFP (tech), $W_t \uparrow$

(d). In equilibrium $L_t = L$.

$$\left.\begin{aligned} Y_t &= K_t^{\alpha} (AL)^{1-\alpha} \\ R_t &= \alpha K_t^{\alpha-1} (AL)^{1-\alpha} \end{aligned}\right\} \text{substitute } L_t \text{ by } L$$

$$W_t = (1-\alpha) K_t^{\alpha} A^{1-\alpha} L^{-\alpha}$$

law of motion of capital.

$$\left\{\begin{aligned} K_{t+1} &= (1-\delta) K_t + I_t \\ &= (1-\delta) K_t + s \cdot Y_t. \\ K_{t+1} &= (1-\delta) K_t + s K_t^{\alpha} (AL)^{1-\alpha} \end{aligned}\right.$$

(e). K^*, steady state, K has no change.

$$K_{t+1} = (1-\delta)K_t + sK_t^{\alpha}(AL)^{1-\alpha}$$

$$K^* = (1-\delta)K^* + sK^{*\alpha}(AL)^{1-\alpha}$$

$$K^* = K^* - \delta K^* + sK^{*\alpha}(AL)^{1-\alpha}$$

$$\delta K^* = sK^{*\alpha}(AL)^{1-\alpha}$$

$$\frac{K^*}{K^{*\alpha}} = \frac{s}{\delta}(AL)^{1-\alpha}$$

$$K^{*(1-\alpha)} = \frac{s}{\delta}(AL)^{1-\alpha}$$

$$K^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}(AL)^{(1-\alpha)\cdot\frac{1}{1-\alpha}}$$

$$K^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}} \cdot AL.$$

c f). $\boxed{K^* = AL\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}}$

elasiticity : $\dfrac{\%\text{ change in } y}{\%\text{ change in } x} = \dfrac{dy/y}{dx/x}$

☆ $\boxed{\dfrac{dy}{y} = \dfrac{1}{y}\cdot dy = \dfrac{d(\log y)}{dy}\cdot dy = d(\log y)}$

$\boxed{\begin{aligned} y &= \log x \\ y' &= \frac{1}{x} \end{aligned}}$

elasticity : $\dfrac{dy/y}{dx/x} = \dfrac{d(\log y)}{d(\log x)}$

$$\underbrace{\log K^*}_{y} = \log(AL) + \frac{1}{1-\alpha}\log\left(\frac{s}{\delta}\right)$$

$$= \underbrace{\log A}_{x} + \underbrace{\log L}_{A} + \underbrace{\frac{1}{1-\alpha}\log s}_{B} - \underbrace{\frac{1}{1-\alpha}\log \delta}_{C}$$

$$\frac{d(\log K^*)}{d(\log A)} = 1$$

$\boxed{\begin{aligned} &y = x \\ &\cancel{y' = 1} \\ &\frac{\partial y}{\partial x} = 1 \end{aligned}}$

(a). $K_t = K_{t+1} = K^*$

$$K^* = \boxed{A_t L_t}\underbrace{\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}}_{\text{Constant.}}$$

$$A_t L_t = \underbrace{A_0 (1+g)^t}_{A_t} \underbrace{L_0 (1+n)^t}_{L_t}$$

$$= A_0 L_0 \left[(1+g)(1+n)\right]^t$$

if $\boxed{\begin{matrix} (1+g)(1+n) = 1 \\ g = n = 0 \end{matrix}}$

$$A_t L_t = A_0 L_0 \cdot 1^t$$

no tech improvement

no pop growth.

(b). $\tilde{k}_t = \frac{K_t}{A_t L_t}$ .... capital per effective worker / labour / people / capita

$k_t = \frac{K_t}{L_t}$ .... capital per worker ... .

K_t ... ~~capital~~ capital.

$$K_{t+1} = (1-\delta) K_t + I_t.$$

$$K_{t+1} = (1-\delta) K_t + s K_t^{\alpha} (A_t L_t)^{1-\alpha}$$

$$\frac{K_{t+1}}{A_t L_t} = (1-\delta) \frac{K_t}{A_t L_t} + \frac{s K_t^{\alpha} (A_t L_t)^{1-\alpha}}{A_t L_t}$$

$$\frac{K_{t+1}}{A_t L_t} = (1-\delta) \tilde{k}_t + s \frac{K_t^{\alpha}}{(A_t L_t)^{\alpha}}$$

$$\frac{K_{t+1}}{A_t L_t} = (1-\delta) \tilde{k}_t + s \cdot \tilde{k}_t^{\alpha}$$

$$\frac{K_{t+1}}{A_t L_t} \cdot \frac{A_{t+1} L_{t+1}}{A_{t+1} L_{t+1}} = \text{....}$$

$A_{t+1} = A_t(1+g)$ $$\frac{K_{t+1}}{A_{t+1} L_{t+1}} \cdot \frac{A_{t+1} L_{t+1}}{A_t L_t} = \text{...}$$

$$\tilde{k}_{t+1} \cdot (1+g)(1+n) = (1-\delta) \tilde{k}_t + s \tilde{k}_t^{\alpha}$$