< saving rate >

Ramsey Model VS Solow model.

Lifetime utility : $U = \sum_{t=0}^{\infty} \beta^t \boxed{\dfrac{C_t^{1-\theta}-1}{1-\theta}}$ CRRA

$u(C_t)$

HH's BC : $C_t + K_{t+1} - (1-\delta)K_t = (1-\tau) Y_t + \underbrace{T_t}_{\text{transfer}}$.

τ is a wedge.

$\tau > 0$ tax.

$\tau < 0$ subsidy $|\tau| < 1$.

Social planner feasibility constraint (resources)

FC.

$$C_t + K_{t+1} - (1-\delta) K_t = Y_t$$

(a) Social planner is optimal.

If $\tau = 0$.

Social planner and HH's problems identical.

(b) Production function $Y = K^{\alpha}(AL)^{1-\alpha}$

$\max_{C_t, K_{t+1}} U$ s.t. BC.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \underbrace{\boxed{\frac{C_t^{1-\theta}-1}{1-\theta}}}_{u(C_t) \to u'(C_t)} + \sum_{t=0}^{\infty} \lambda_t [(1-\tau)K_t^{\alpha}(AL)^{1-\alpha} + T_t - C_t - K_{t+1} + (1-\delta) K_t]$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t C_t^{-\theta} - \lambda_t = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \lambda_{t+1}[\alpha(1-\tau)K_{t+1}^{\alpha-1}(AL)^{1-\alpha} + 1 - \delta] = 0 \quad (2)$$

$\frac{\partial \mathcal{L}}{\partial \lambda_t}$ / BC. $(1-\tau)K_t^{\alpha}(AL)^{1-\alpha} + T_t - C_t - K_{t+1} + (1-\delta)K_t = 0$ (3)

$$\lambda_t = \beta^t u'(C_t) \qquad \lambda_{t+1} = \beta^{t+1} u'(C_{t+1})$$

$$①: \quad \lambda_t = \beta^t C_t^{-\theta} \Rightarrow \lambda_{t+1} = \beta^{t+1} C_{t+1}^{-\theta}$$

$$-\beta^t C_t^{-\theta} + \beta^{t+1} C_{t+1}^{-\theta} [(1-\tau)\alpha K_{t+1}^{\alpha-1}(AL)^{1-\alpha} + 1 - \delta] = 0.$$

fill by yourself.

$$\frac{u'(C_t)}{u'(C_{t+1})} = \ldots\ldots$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\theta} = \beta[1 + (1-\tau)\alpha K_{t+1}^{\alpha-1}(AL)^{1-\alpha} - \delta]$$

…… Euler equation

$$u'(C_t) = \underset{(0,1)}{\beta}[1 + (1-\tau)\alpha \underbrace{K_{t+1}^{\alpha-1}(AL)^{1-\alpha}}_{\text{MPK}} - \delta]\, u'(C_{t+1})$$

(c) C_t K_{t+1} Y_t T_t.

$t=0$.

HH's BC $\quad (1-\tau)K_0^{\alpha}(AL)^{1-\alpha} + T_0 - C_0 - K_1 + (1-\delta)K_0 = 0.$

Euler $\quad \left(\frac{C_1}{C_0}\right)^{\theta} = \beta[1 + (1-\tau)\alpha K_1^{\alpha-1}(AL)^{1-\alpha} - \delta]$

GDP $\quad Y_0 = K_0^{\alpha}(AL)^{1-\alpha}$

Transfer $\quad T_0 = \tau Y_0.$

4 equations 5 unknowns.

(d) BC: $(1-\tau)K_t^{\alpha}(AL)^{1-\alpha} + T_t - C_t - K_{t+1} + (1-\delta)K_t = 0$

$T_t = \tau Y_t$

$= \tau K_t^{\alpha}(AL)^{1-\alpha}$

Euler: $\left(\frac{C_{t+1}}{C_t}\right)^{\theta} = \beta[1 + (1-\tau)\alpha K_{t+1}^{\alpha-1}(AL)^{1-\alpha} - \delta]$

(e) $g_t^C = \frac{C_t}{C_{t-1}} - 1 \Rightarrow \frac{C_t}{C_{t-1}} = g_t^C + 1$

$(g_t^C + 1)^{\theta} = \beta[1 + (1-\tau)\alpha \uparrow K_t^{\alpha-1}(AL)^{1-\alpha} - \delta] \downarrow$

if $K_t \uparrow$ $(\alpha - 1 < 0)$

MPK $\downarrow$.

I $\downarrow$ S today $\downarrow$ C today $\uparrow$.

$g_t^C \downarrow$.

(f) Steady state $\quad K_{t+1} = K^*$ $\quad C_{t+1} = C_t = C^*$

$$\left(\frac{C^*}{C^*}\right)^{\theta} = \beta\left[1 + (1-\tau)\alpha K^{*\,\alpha-1}(AL)^{1-\alpha} - \delta\right]$$

fill by yourself.

$$K^* = \left(\frac{\beta(1-\tau)\alpha}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} AL.$$

$$Y^* = K^{*\alpha}(AL)^{1-\alpha}$$

$$= \left(\frac{\beta(1-\tau)\alpha}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} AL.$$

(g). $K_{t+1} = (1-\delta)K_t + I_t$ $\quad \cdots$ Law of motion of capital.

$$I^* = K^* - (1-\delta)K^*$$

$$= \delta\left(\frac{\beta(1-\tau)\alpha}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} AL.$$

$I^* = sY^*$

$$s^* = \frac{I^*}{Y^*} = \frac{\delta\Big(\overbrace{\frac{\beta(1-\tau)\alpha}{1-\beta(1-\delta)}}^{B}\Big)^{\frac{1}{1-\alpha}} \cancel{AL}}{\Big(\underbrace{\frac{\beta(1-\tau)\alpha}{1-\beta(1-\delta)}}_{B}\Big)^{\frac{\alpha}{1-\alpha}} \cancel{AL}}$$

$$\cancel{\frac{1}{1-\alpha}\cdot\frac{1-\alpha}{\alpha} = \frac{1}{\alpha}}$$

$$= \delta B^{\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}}$$

$$= \delta B^{\frac{1-\alpha}{1-\alpha}}$$

$$= \delta B.$$

(h) Ramsey vs Solow.

S is endogenous from HH. S is exogeneous.

(i) $C^* = Y^* - I^*$

do by yourself.

(j) $C^*(\tau)$

$$\frac{\partial C^*(\tau)}{\partial \tau} = AL\left(\frac{\beta\alpha}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\right)(1-\tau)^{\frac{\alpha}{1-\alpha}-1}\frac{\partial(1-\tau)}{\partial\tau}$$

$$- AL\delta\left(\frac{\beta\alpha}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{1-\alpha}\right)(1-\tau)^{\frac{1}{1-\alpha}-1}\frac{\partial(1-\tau)}{\partial\tau}$$

fill by yourself

$$\boxed{\tau = - \frac{1-\beta^{+}}{\delta\beta_{+}}}$$

$\tau < 0$. Subsidy.