$$\frac{C_t^{1-\alpha}}{1-\alpha} \quad (CRRA)$$

$u(C_t)$ → Shadow price.

Q1

(a) $\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \log C_t + \sum_{t=0}^{\infty} \lambda_t [K_t^{\alpha}(A_t L_t)^{1-\alpha} - C_t - K_{t+1} + (1-\delta) K_t]$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t C_t^{-1} - \lambda_t = 0 \quad ①.$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \lambda_{t+1} [\alpha K_{t+1}^{\alpha-1} (A_t L_t)^{1-\alpha} + 1 - \delta] = 0 \quad ②$$

①: $\Rightarrow \lambda_t = \beta^t C_t^{-1} \qquad \lambda_t = \beta^t u'(C_t)$

②: $\Rightarrow \dfrac{\lambda_t}{\lambda_{t+1}} = \alpha K_{t+1}^{\alpha-1} (A_t L_t)^{1-\alpha} + 1 - \delta$

using ① substitute λ_t, λ_{t+1} in ②.

$$\frac{\beta^t C_t^{-1}}{\beta^{t+1} C_{t+1}^{-1}} = \alpha K_{t+1}^{\alpha-1} (A_t L_t)^{1-\alpha} + 1 - \delta.$$

$$\frac{C_{t+1}\uparrow}{C_t} = \beta \cdot [\alpha \overset{\uparrow}{K_{t+1}^{\alpha-1}} (A_t L_t)^{1-\alpha} + 1 - \delta].$$

$K\downarrow$ ↳ Euler equation.

$$\frac{u'(C_t)}{u'(C_{t+1})} = (\cdots)$$

$$u'(C_t) = \beta \cdot [\alpha K_{t+1}^{\alpha-1} (A_t L_t)^{1-\alpha} + 1 - \delta] \cdot u'(C_{t+1})$$

(b). BGP: $\frac{C^*_{t+1}}{C^*_t} = 1+g$.

$$1+g = \beta [\alpha K^{*\alpha-1}_{t+1} (A_{t+1} L_{t+1})^{1-\alpha} + 1 - \delta].$$

$$\frac{1+g}{\beta} = [\alpha \cdots\cdots + 1 - \delta]$$

$$\frac{1+g-\beta(1-\delta)}{\alpha\beta} = K^{*\alpha-1}_{t+1} (A_{t+1} L_{t+1})^{1-\alpha}$$

Since. $K^*_{t+1} = (1+g) K^*_t$

$A_{t+1} = (1+g) A_t$

$L_{t+1} = L$. (drop time index here)

① one period backward.

$$\frac{1+g-\beta(1-\delta)}{\alpha\beta} = [(1+g) K^*_t]^{\alpha-1} [(1+g) A_t]^{1-\alpha} L^{1-\alpha}$$

$$= K^{*\alpha-1}_t A_t^{1-\alpha} L^{1-\alpha}$$

② . $Y^*_t = K^{*\alpha}_t (A_t L)^{1-\alpha}$

$$\left(\frac{1+g-\beta(1-\delta)}{\alpha\beta}\right)^{\frac{1}{\alpha-1}} = K_t^{*(\alpha-1)\cdot\frac{1}{\alpha-1}} (A_t L)^{1-\alpha\cdot\frac{1}{\alpha-1}}$$

$$= K^*_t (A_t L)^{-1}$$

$$(\cdots\cdots)^{\frac{\alpha}{\alpha-1}} = K^{*\alpha}_t (A_t L)^{-\alpha}$$

$$(\cdots\cdots)^{\frac{\alpha}{\alpha-1}} A_t L = K^{*\alpha}_t (A_t L)^{1-\alpha}$$

$$\left(\frac{\alpha\beta}{1+g-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} \cdot A_t L = Y^*_t$$

(c) $K_0 = K^*$ (not effected)

output gap is purely from labor supply shortage.

$L = 1$. $\frac{Y_0}{Y_0'} = \frac{K_0^\alpha (A_0 \theta)^{1-\alpha}}{K_0^\alpha (A_0 \cdot 1)^{1-\alpha}} = \theta^{1-\alpha} < 1$.

(d). Period 0. $Y_0 < Y^*$

$Y_0 = C_0 + I_0$.

3 scenarios:

① $I_0 = I^*$, $C_0 < C^*$ } contradictions.

② $C_0 = C^*$, $I_0 < I^*$.

③ $C_0 < C^*$, $I_0 < I^*$. ✓

①. $K_1 = (1-\delta)K_0 + I_0 = K^*$ } nothing happened? wrong!

$\Rightarrow C_1 = C^*$

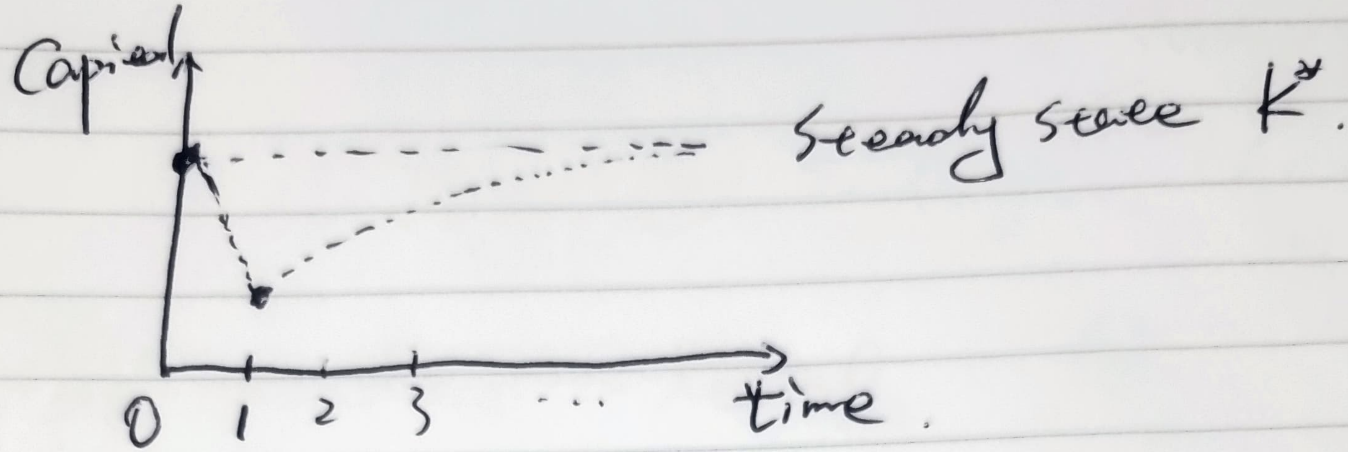
②. $K_1 < K^*$ from law of motion of K.

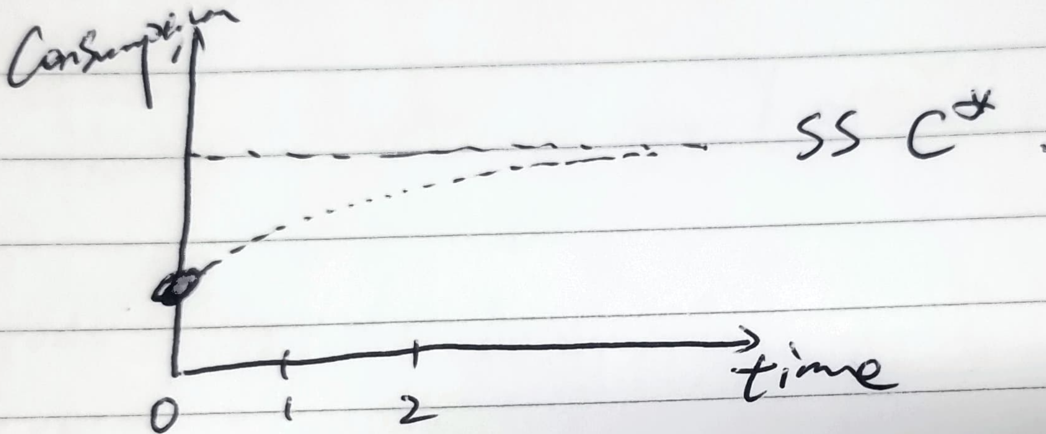
$\Rightarrow C_1 > C_0 = C^*$ from Euler rule

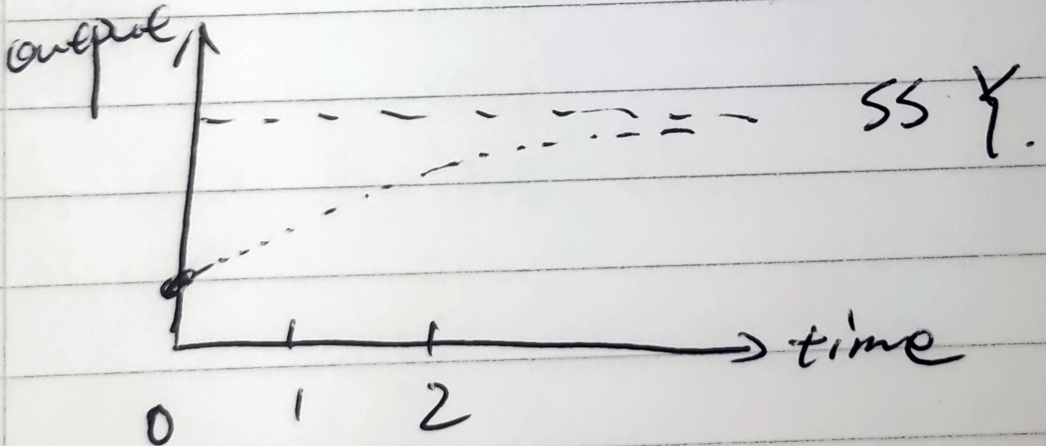
but in reality, if $K_1 < K^*$.

need saving more, reduce C.

③. ✓.


Capital
Steady state K^*.
0
1
2
3
...
time.


Consumption
SS C^*.
time
0
1
2


output
SS Y.
time
0
1
2